TRIGONOMETRY PROBLEMS

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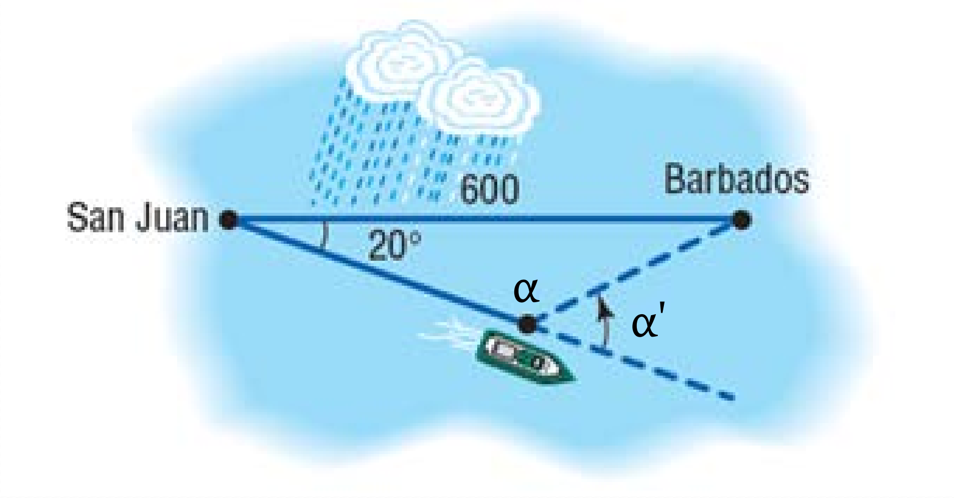
Problem 1

A cruise ship maintains an average speed of 15 knots in going from San Juan, Puerto Rico, to Barbados, West Indies, a distance of 600 nautical miles. To avoid a tropical storm, the captain heads out of San Juan in a direction of 20 degrees off a direct heading to Barbados. The captain maintains the 15 knot speed for 10 hours, after which time the path to Barbados becomes clear of storms.

a) Through what angle should the captain turn to head directly to Barbados?

b) Once the turn is made, how long will it be before the ship reaches Barbados if the same 15-knot speed is maintained?



Information

Distance between San Juan and Barbados: $a = 600$ nautical miles

$v = 15$ knots $= 27,78$ km/h

$\beta = 20°$

$\Delta t = 10h$

$\alpha' = ?$

Solution

A)

Distance between San Juan and the ship: $c = 15$ kn x 10 h = 150 nautical miles

Distance between the Ship and Barbados:

$b = \sqrt{600^2 + 150^2 - 2*600*150*\cos 20°} = 462$ nautical miles (Law of cosines)

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow \sin\alpha = \frac{\sin\beta}{b} * a = \frac{\sin 20°}{462} * 600 = 0.444$ (Law of sines)

$\alpha = 180° - \sin^{-1}(0.444) = 180° - 26.4° = 153.6°$ (α must be obtuse)

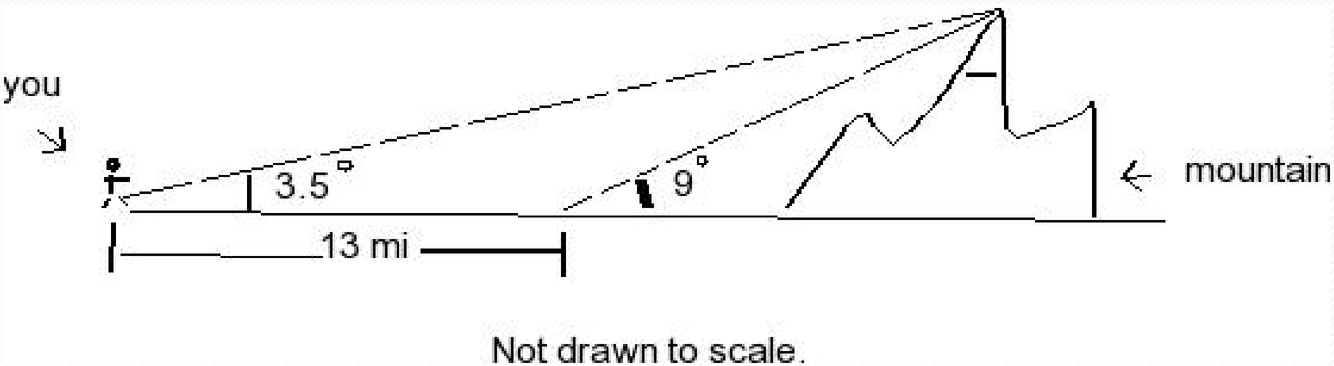
$\alpha' = 180° - \alpha = 26.4°$

B) Ship - Barbados (time) = 462 nmi ÷ 15 kn = 30.8 h

Problem 2

In traveling across flat land you notice a mountain directly in front of you. Its angle of elevation (to the peak) is 3.5° . After you drive 13 miles closer to the mountain, the angle of elevation is 9°.

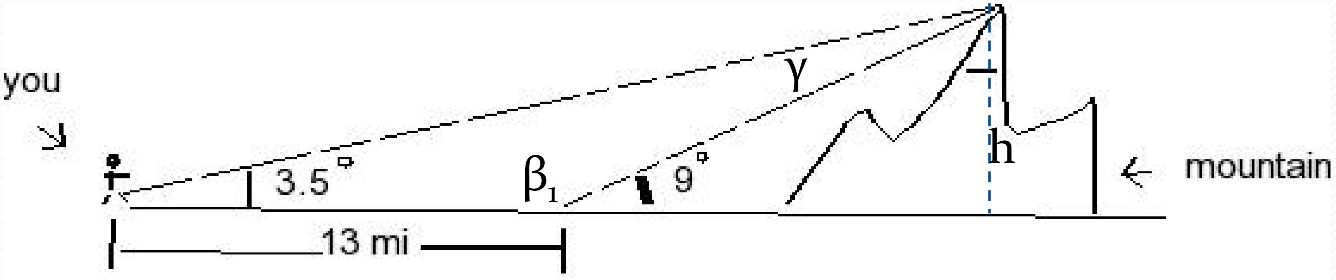
Approximate the height of the mountain.



Information

α = 3.5°
β = 9°

Height = ?



Not drawn to scale.

Solution

$\beta_1 = 180° - 9° = 171°$
$\gamma = 180° - 171° - 3.5° = 5.5°$

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} \Rightarrow a = \frac{c}{\sin\gamma}\sin\alpha = \frac{13 \text{ miles}}{\sin 5.5°}\sin 3.5° = 8.28 \text{ miles}$$

$h = a * \sin\beta = 8.3 \text{ miles} \times \sin 9° = 1.295 \text{ miles}$